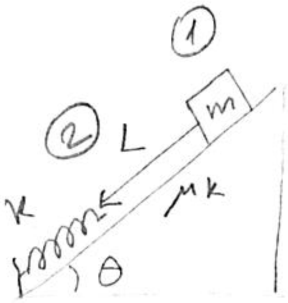
1. A block of mass m starts from rest and slides a distance L down an incline of angle θ, where it contacts an unstressed spring of negligible mass as in the figure. The coefficient of kinetic friction between the block and the incline is μ_k. The mass slides an additional distance x as it is brought momentarily to rest by compressing the spring (with spring constant k). In terms of the given values,
a.) What is the speed of the mass just before it reaches the spring?
b.) What is the maximum compression d of the spring?



$$W_{f_k} = (K_2 - K_1) + (U_2 - U_1)$$

$$W_{f_k} = -\mu_k mg \cos\theta\, L$$

$$U_1 = mgL\sin\theta \quad , \quad K_1 = 0$$

$$U_2 = 0 \quad , \quad K_2 = \frac{1}{2}mv^2$$

a.) Speed before the block hits the spring

$$-\mu_k mg\cos\theta\, L = \left(\frac{1}{2}mv^2 - 0\right) + (0 - mgL\sin\theta)$$

$$\cancel{m}gL\sin\theta - \mu_k \cancel{m}g\cos\theta\, L = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{2gL(\sin\theta - \mu_k\cos\theta)}$$

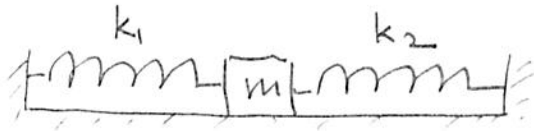
b.) Apply the equ. to points 1 and 3 (when the spring is compressed an amount of d)

$$U_1 = mg(L+d)\sin\theta \quad , \quad K_1 = 0 \qquad W_{frict} = (K_3 - K_1) + (U_3 - U_1)$$

$$U_3 = \frac{1}{2}kd^2 \qquad K_3 = 0$$

$$-\mu_k mg\cos\theta(L+d) = 0 + \left(\frac{1}{2}kd^2 - mg(L+d)\sin\theta\right)$$

2. A block of mass m is connected to two ideal horizontal springs of spring constants k_1 and k_2. The system is initially in equilibrium on a horizontal, frictionless surface. The block is pushed a distance x from its equilibrium position in either direction and released from rest. Find the speed of the block as it passes from its equilibrium position. (in terms of given quantities).



$$K_i + U_i = K_f + U_f$$

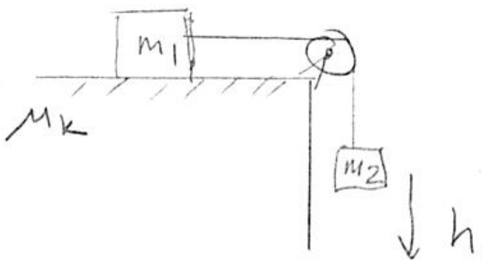
$K_i = 0$ $\qquad$ $U_f = 0$

$U_i = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x^2$ $\qquad$ $K_f = \frac{1}{2} m v^2$

$$\frac{1}{2}(k_1 + k_2) x^2 = \frac{1}{2} m v^2$$

$$V = \sqrt{\frac{(k_1 + k_2) x^2}{m}}$$

3.) The coefficient of kinetic friction between the block m_1 and the surface is μ_k. The system starts from rest. In terms of the given values, find the speed of m_2 when it has fallen a distance of h.

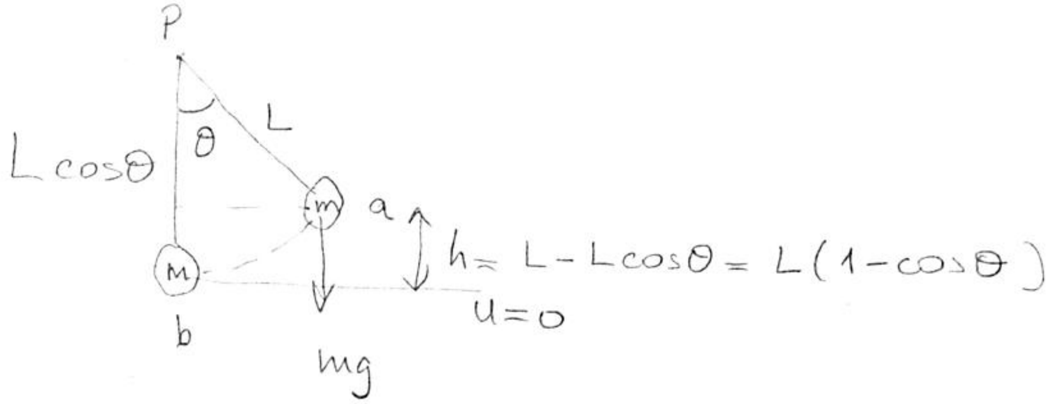


$$W_{friction} = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 - m_2 g h$$

$$-\mu_k m_1 g h = \frac{1}{2}(m_1 + m_2) v_f^2 - m_2 g h$$

$$v_f = \sqrt{\frac{2}{m_1 + m_2}(m_2 g h - \mu_k m_1 g h)}$$

4. A pendulum consists of a sphere of mass m attached to a light cord of length L. The sphere is released from rest when the cord makes an angle θ with the vertical. Find the speed of the sphere when it is at the lowest point, b in terms of given quantities.



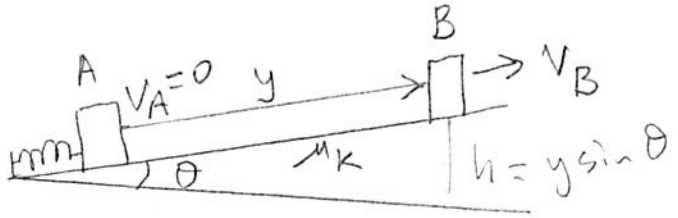
$$K_b = \frac{1}{2} m v_b^2 \quad K_a = 0 \quad , \quad K_b = \frac{1}{2} m V_b^2$$

$$U_a = mgh \qquad U_b = 0$$

$$\frac{1}{2} m V_b^2 + 0 = 0 + mg\,L(1 - \cos\theta)$$

$$V_b = \sqrt{2gL(1 - \cos\theta)}$$

5. A block with mass m is placed against a compressed spring at the bottom of an incline with angle θ at point A. When the spring is released, it projects the block up the incline. At point B, a distance of y up the incline from A, the block's velocity is v_B and is no longer in contact with the spring. The coefficient of kinetic friction between the block and the incline is μ_K. The mass of the spring is negligible. Calculate the amount of potential energy that was initially stored in the spring in terms of the given quantities.



$$W_{friction} = (K_B - K_A) + (U_B - U_A)$$

$$W_{friction} = -\mu_K mg\cos\theta\, y$$

$$K_A = 0 \quad , \quad K_B = \frac{1}{2} m v_B^2$$

$$U_A = U_{A\,elastic} \quad , \quad U_B = mgy\sin\theta$$

$$-\mu_K mg\cos\theta\, y = \left(\frac{1}{2} m v_B^2 - 0\right) + \left(mgy\sin\theta - U_{A\,elastic}\right)$$

$$U_{A\,elastic} = \frac{1}{2} m v_B^2 + mgy\sin\theta + \mu_K mg\cos\theta\, y$$